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ONE-DIMENSIONAL LASER HEATING OF A STATIONARY PLASMA

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16. Abstract

The problem of laser energy addition to a nonuniform one-dimensional plasma is considered. The object is to study plasmas which may achieve temperatures and densities of interest for application to controlled thermonuclear reactions (CTR). The advent of the nanosecond and picosecond lasers makes it possible to consider the heating of stationary, non-thermally conducting plasmas with a nonuniform density distribution in which only the electrons are heated. The solutions show that for a wide range of relevant parameters these assumptions are valid. The temperatures that can be produced are of interest in CTR. Depending on the steepness of the density gradient, either wave-like or simultaneous heating is produced. Also, the temperature distribution is very much nonuniform and can significantly affect the subsequent dynamics of the plasma.

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I. INTRODUCTION

The advent of high power pulsed lasers now makes it possible to heat a dense plasma to extremely high temperatures. Because of the laser's ability to create plasmas that are both very hot and dense, this type of heating is an attractive approach to achieving a controlled thermonuclear reaction (CTR). However, an important limitation on the maximum temperature that can be created arises from the plasma motion. During the laser pulse the plasma may begin to expand and much of the thermal energy would be converted to directed kinetic energy of the plasma motion, as was first noted by Basov[1] and Dawson[2].

The recent development of powerful sub-nanosecond Q-switched lasers and mode-locked picosecond lasers makes it possible to overcome the problem of expansion during the heating process. For appropriately short laser pulses, the plasma can be heated to thermonuclear temperatures before significant macroscopic fluid motion begins. The irradiation of a plasma by these short laser pulses then becomes a stationary heating problem.

A study has been made of the stationary heating of a one-dimensional plasma with an arbitrary density profile. The purpose of the study was to determine the fine details of the heating process as well as the overall features. The spatial distribution of temperature is one detail of interest which will hold the key to the plasma motion that will follow the heating process. Another detail of interest is the fraction of laser energy that is lost by reflection from the nonuniform plasma.

Several theoretical investigations of laser plasma heating have been made; but most models have neglected the fine details and concentrated on over-all results, e.g., the temperature achieved. Previous investigations have also used time scales in which plasma motion is significant.

A study by Fader[1] considered the expansion of a spherically symmetric plasma subject to Q-switched laser radiation. The expansion of the plasma was found to be a major limitation on the temperatures achieved according to both numerical and analytical calculation. This study assumed spatially uniform absorption throughout the plasma.

A study by Kidder[2] also considered the motion of a spherically symmetric plasma but with irradiation by a radially convergent light pulse. His numerical study also showed the limitation of expansion on the temperatures achieved.

Dawson, et al., studied the over-all absorption characteristics of a one-dimensional nonuniform overdense plasma.[3] Without studying the details of absorption, they found the length of plasma necessary to adequately absorb the radiation.

This paper also uses a one-dimensional nonuniform plasma and considers the entire heating process for time scales sufficiently short that plasma motion can be neglected. The results indicate that for a thick underdense region, a heating wave propagates into the plasma; and for a thin underdense region, the heating is simultaneous-- with an extreme hot spot appearing at the critical density (where the plasma frequency equals the laser frequency). The heating wave phenomena has been mentioned before by Zel'dovich and Raiser[4] and arose in the numerical studies of Kidder[2] and the analytical studies of Rehm.[5]

Another result is that maximum temperature or other optimizing conditions can be achieved by proper tailoring of the initial density profile and plasma size. The idea of tailoring the plasma to optimize the results was first suggested by Daiber, et al.,[6] and later in another scheme by Lubin.[7]

II. MODEL

The plasma is taken to be a one-dimensional nonuniform fully ionized gas being irradiated by a laser of wavelength λ_o at normal incidence. Figure 1 is a diagram of the configuration. The initial density and temperature profiles are arbitrary to the extent that a fully ionized gas is a valid assumption. The plasma is assumed to be a dense ideal gas mixture of electrons and ions such that there is charge neutrality at every point for all times.

The one-dimensional two-temperature continuum equations will be used with the addition of terms accounting for radiative energy addition. The electromagnetic force will be neglected since its effect is small compared to the effect of thermal forces. Also, a form of the radiative transfer equation is added which neglects thermal and bremsstrahlung radiation. The radiative transfer equation is simplified to account only for laser radiation entering the plasma (+ x direction) and does not take into account the reflection of radiation that would occur if the plasma is overdense (where the plasma frequency equals the laser frequency). If the laser radiation does penetrate to this "critical density," then appropriate calculations must be made to account for the reflected light. The absorption coefficient used is that of inverse bremsstrahlung.

If the plasma is overdense, then $x = R$ will define the point at which the critical density ρ_c occurs. If the plasma is not overdense at any point, then $x = R$ gives the rear edge of the plasma.

Non-Dimensionalization Scheme

The length scale used is R . The time scale is the laser pulse length t_p since only the heating process is considered in this paper. Hence, the independent variables, dimensionless distance and time will always be of order one or less. The temperature scale used for both electron and ion temperature is T_o and is defined as the temperature change that would arise if all the energy in the laser pulse were added uniformly to a critical density plasma of thickness R . Then

$$T_o = \frac{J_o/A}{{}^3/_2 k n_{ec} R} \tag{1}$$

where J_o/A is the energy contained in the laser pulse per unit area of plasma, k is the Boltzmann constant and n_{ec} is the critical electron density (where plasma frequency equals laser frequency) given by

$$n_{ec} = \frac{4\pi^2 \varepsilon_o m_e c^2}{\lambda_o^2 e^2} \tag{2}$$

The density scale is the critical density, i.e., the mass density which corresponds to the critical electron density, $\rho_c = m_i n_{ec}/Z$. The pressure scale p_o is the pressure that would arise in a perfect gas at temperature T_o and density ρ_c. The velocity scale is the acoustic speed for a perfect gas at p_o and ρ_c, i.e., $a_o^2 = 5p_o/3\rho_c$. The electron and ion entropies are scaled using the constant volume specific heat. The absorption coefficient, electron-ion equilibration time, and the electron and ion thermal conduction coefficients are taken from Spitzer.[9] These are all based upon a scale using a density ρ_c and a temperature T_o ;

$$\kappa_o = \text{constant } \rho_c^2/T_o^{3/2} \tag{3}$$

$$t_{eq_o} = \text{constant } T_o^{3/2}/\rho_c \tag{4}$$

$$K_{e,i_o} = \text{constant}_{e,i}\ T_o^{5/2} \tag{5}$$

The radiation intensity scale is the average laser intensity during the pulse.

$$I_o = J_o/At_p \tag{6}$$

Summarizing, the dimensionless variables are:

Distance	$y = x/R$	Absorption coefficient	$K = \kappa/\kappa_o$
Time	$\tau = t/t_p$	Fluid velocity	$U = u/a_o$

Density	$\zeta = \rho/\rho_c$	Electron entropy	$\sigma_e = S_e/C_v$
Pressure	$\Pi = p/p_o$	Ion entropy	$\sigma_i = S_i/C_v$
Electron temperature	$\theta_e = T_e/T_o$	Intensity	$i = I/I_o$
Ion temperature	$\theta_i = T_i/T_o$		(7)

Parameters

Using the non-dimensionalization scheme described, five parameters arise which control the plasma behavior. The first parameter is

$$\varepsilon = a_o t_p/R \tag{8}$$

ε is the ratio of the laser pulse time to the time for an acoustic wave to traverse the underdense region. If $\varepsilon << 1$, then the pulse length is so short that essentially no plasma motion occurs. If $\varepsilon \geq O(1)$, then significant motion occurs before the pulse ends.

$$\alpha = \kappa_o R \tag{9}$$

α is the ratio of the thickness of the underdense region to the absorption length scale. The absorption length is the reciprocal of the absorption coefficient. If $\alpha << 1$, then the plasma is nearly transparent to the radiation. If the plasma is overdense, then the radiation will be reflected and the reflected beam will be negligibly reduced in intensity. The result will be to effectively double the absorption coefficient. If $\alpha >> 1$, then the plasma is nearly opaque to the radiation.

$$\lambda = t_p/t_{eq_o} \tag{10}$$

λ is the ratio of the pulse time to the electron-ion equilibration time. If $\lambda << 1$, then the electrons will tend to be heated by inverse bremsstrahlung; but the ions will remain nearly frozen during the laser pulse. If $\lambda >> 1$, then the electron and ion temperatures will be tha same.

$$\eta_{e,i} = t_p K_{e,i_o}/\rho_c C_v R^2 \tag{11}$$

$\eta_{e,i}$ is (for electrons and ions respectively) the ratio of pulse time to the time for a thermal diffusion wave to traverse the underdense region. Then, for example, if $\eta_e << 1$, there will be very little thermal conduction by the electrons during the laser pulse. If $\eta_e >> 1$, the thermal conduction will be strong, tending to equalize the temperature of the plasma.

Equations

The governing equations in dimensionless form are

$$\frac{\partial\zeta}{\partial\tau} + \varepsilon \frac{\partial\zeta U}{\partial y} = 0 \tag{12}$$

$$\frac{\partial U}{\partial\tau} + \varepsilon\left[U\frac{\partial U}{\partial y} + \frac{3}{5}\,\frac{1}{\zeta}\,\frac{\partial\Pi}{\partial y}\right] = 0 \tag{13}$$

$$\frac{\partial\sigma_e}{\partial\tau} + \varepsilon U\frac{\partial\sigma_e}{\partial y} = \alpha K\frac{i}{\zeta\theta_e} - \lambda\frac{\zeta}{\theta_e^{3/2}}\,\frac{\theta_e - \theta_i}{\theta_i} + \eta_e\frac{2/7}{\zeta\theta_e}\,\frac{\partial^2\theta_e^{7/2}}{\partial y^2} \tag{14}$$

$$\frac{\partial\sigma_i}{\partial\tau} + \varepsilon U\frac{\partial\sigma_i}{\partial y} = \lambda\frac{\zeta}{\theta_e^{3/2}}\,\frac{\theta_e - \theta_i}{\theta_e} + \eta_i\frac{2/7}{\zeta\theta_i}\,\frac{\partial^2\theta_i^{7/2}}{\partial y^2} \tag{15}$$

$$\frac{\partial i}{\partial y} + \alpha K i = 0 \tag{16}$$

$$K = \frac{\zeta^2}{\theta_e^{3/2}\sqrt{1-\zeta}} \tag{17}$$

$$\Pi = \zeta[\theta_e + \frac{1}{Z}\theta_i] \tag{18}$$

$$\theta_e = \zeta^{2/3}e^{\sigma_e} \tag{19}$$

$$\theta_i = \zeta^{2/3}e^{\sigma_i} \tag{20}$$

The equation for absorption coefficient (17) breaks down for densities very near the critical density ($\zeta = 1$). There is actually an extremely high value of the absorption coefficient at that point, but not an infinity. Hence, solutions that are found will probably have a singularity at that point and thus cannot be trusted for ζ very near one. This singularity will usually be integrable so that integrations with respect to y over the singularity will describe the actual case fairly accurately.

The initial conditions will be an initial temperature and an initial density profile. The boundary conditions will be on the intensity; the intensity entering the plasma will be $d\Phi/d\tau$ where Φ is the fraction of the laser energy delivered up to time τ ;

$$\Phi = \frac{1}{J_o/A} \int_o^t I(t)dt \tag{21}$$

so that $\Phi = 0$ at $\tau = 0$ and $\Phi = 1$ at $\tau = 1$ (the end of the pulse).

III. EQUATIONS FOR A STATIONARY FROZEN NON-THERMALLY CONDUCTING PLASMA

For sufficiently rapid energy addition, the plasma may be significantly heated before the random thermal energy begins to change appreciably to ordered fluid motion. The heating may be rapid enough that the diffusing effect of thermal conductivity will not have a significant effect. Also, with sufficiently rapid heating, the equilibration processes whereby the heated electrons transfer their energy to the ions will not have proceeded to any significant extent.

The problem studied in this paper is the case where all of these conditions appear simultaneously. Then the heating is essentially a stationary, non-thermally conducting process whereby energy is transferred from a beam of light to the electrons in the plasma. No energy is transferred between electrons by thermal conduction, and no energy is transferred to the ions, either by electron-ion equilibration or by ordered plasma motion.

The cases in which plasma motion, thermal conduction, and electron-ion equilibration become significant during the laser pulse form problems for later study. Another interesting problem for later study is the behavior of the plasma <u>after</u> being heated in a stationary, non-thermally conducting, nonequilibrium manner (as studied in this paper).

The requirement for a stationary plasma is that $\varepsilon << 1$, and for a non-conducting plasma, $\eta_{e,i} << 1$. Also, for a plasma in which the electron temperature is large compared to the ion temperature, then $\lambda\theta_e/\theta_i << 1$ is required.

For the limit ε, λ, $\lambda\theta_e/\theta_i$, $\eta_{e,i} \to 0$, the governing equations simplify considerably. The continuity equation (12) becomes $\partial\zeta/\partial\tau = 0$ so that the density profile remains stationary $\zeta = \zeta(y)$. The momentum equation (13) becomes $\partial U/\partial\tau = 0$ so that the velocity remains zero throughout the process. The electron entropy equation (14) becomes

$$\frac{\partial\sigma_e}{\partial\tau} = \frac{\alpha K}{\zeta}\,\frac{i}{\theta_e} \qquad (22)$$

The radiative transfer equation (16) remains the same, as does the absorption coefficient (17), and all three equations of state (18), (19), and (20). The electron entropy equation (22) plus the radiative transfer equation (16), the absorption coefficient (17), and one equation of state (19) form a complete set of four equations in four unknowns.

These can be reduced to a single equation for electron temperature. Solving (19) for σ_e, taking the time derivative and eliminating $\partial\sigma_e/\partial\tau$ using (22) yields

$$\frac{\partial\theta_e}{\partial\tau} = \frac{\alpha K}{\zeta}\, i \qquad (23)$$

Then solving for i and using (17)

$$i = \frac{2}{5}\,\frac{\sqrt{1-\zeta}}{\alpha\zeta}\,\frac{\partial\theta_e^{5/2}}{\partial\tau} \qquad (24)$$

Differentiating with respect to y

$$\frac{\partial i}{\partial y} = \frac{2}{5}\,\frac{\sqrt{1-\zeta}}{\alpha\zeta}\,\frac{\partial^2\theta_e^{5/2}}{\partial y\partial\tau} - \frac{2}{5}\,\frac{1-\frac{1}{2}\zeta}{\alpha\zeta^2\sqrt{1-\zeta}}\,\frac{d\zeta}{dy}\,\frac{\partial\theta_e^{5/2}}{\partial\tau} \qquad (25)$$

Combining (24) and (25) using (16) to eliminate i and (17) to eliminate K yields

$$0 = \frac{2}{5}\,\frac{\partial^2\theta_e^{5/2}}{\partial y\partial\tau} - \frac{2}{5}\,\frac{1-\frac{1}{2}\zeta}{\zeta(1-\zeta)}\,\frac{d\zeta}{dy}\,\frac{\partial\theta_e^{5/2}}{\partial\tau} + \frac{\alpha\zeta^2}{\sqrt{1-\zeta}}\,\frac{\partial\theta_e}{\partial\tau}$$

which can be immediately integrated once on τ to give

$$\frac{2}{5}\,\frac{\partial\theta_e^{5/2}}{\partial y} - \frac{2}{5}\,\frac{1-\frac{1}{2}\zeta}{\zeta(1-\zeta)}\,\frac{d\zeta}{dy}\,\theta_e^{5/2} + \frac{\alpha\zeta^2}{\sqrt{1-\zeta}}\,\theta_e = f(y) \qquad (26)$$

f(y) arises out of integrating with respect to τ and is determined by the initial conditions. If the initial condition is $\theta_e(y,0) = \theta_e^{\circ}(y)$, then (26) becomes

$$0 = \frac{2}{5}\frac{\partial}{\partial y}[\theta_e^{5/2} - \theta_e^{o5/2}] - \frac{2}{5}\frac{1 - \frac{1}{2}\zeta}{\zeta(1 - \zeta)}\frac{d\zeta}{dy}[\theta_e^{5/2} - \theta_e^{o5/2}]$$

$$+ \frac{\alpha\zeta^2}{\sqrt{1 - \zeta}}[\theta_e - \theta_e^o] \qquad (27)$$

This is the equation governing electron temperature for a stationary, frozen, non-thermally conducting plasma.

It is noteworthy that although the dependent variable θ_e is a function of both y, and τ, (27) is essentially an ordinary differential equation in θ_e in that only the partial derivative with respect to y appears. The only precaution in considering (27) as an ordinary differential equation is that in the solution, arbitrary functions of τ arise instead of constants (as in the case of an ordinary differential equation).

In order to formulate a well posed problem, appropriate boundary and initial conditions must accompany the governing equation (27). The initial condition is

$$\theta_e(y,0) = \theta_e^o(y) \qquad (28)$$

The density profile $\zeta(y)$ remains the same throughout the stationary process, and hence is the same as the initial density profile.

The basic boundary condition is that the intensity at the outer edge of the plasma equals the intensity delivered by the laser,

$$i(0,\tau) = d\Phi/d\tau$$

But a boundary condition on temperature θ_e must be developed. Equations (17) and (23) combine to give

$$\frac{\partial\theta_e^{5/2}}{\partial\tau} = \frac{5}{2}\frac{\alpha\zeta}{\sqrt{1 - \zeta}}\, i$$

and the boundary condition is

$$\left.\frac{\partial\theta_e^{5/2}}{\partial\tau}\right|_{y=0} = \frac{5}{2}\frac{\alpha\zeta(0)}{\sqrt{1 - \zeta(0)}}\frac{d\Phi}{d\tau} \qquad (29)$$

If $\zeta(y)$ is such that $\zeta(0) = 0$, then the appropriate boundary condition is on the first derivative with respect to y

$$\left.\frac{\partial^2 \theta_e^{5/2}}{\partial y \partial \tau}\right|_{y=0} = \frac{5}{2} \alpha \left.\frac{d\zeta}{dy}\right|_{y=0} \frac{d\Phi}{d\tau} \tag{30}$$

for $\zeta(0) = 0$, and $d\zeta/dy\big|_{y=0} \neq 0$. Equations (28), and (29) or (30) together with the governing differential equation (27) form a well posed problem.

IV. DISCUSSION OF PARAMETERS

It is important to know when a stationary frozen non-thermally conducting plasma actually arises, i.e., under what conditions ε, λ, $\eta_{e,i} \ll 1$. Certainly not all regimes of interest will satisfy these criteria.

One of the factors determining these parameters is the size and density of the plasma. For this discussion, the plasma is assumed to be overdense (i.e., the plasma frequency at the point of highest density is greater than the laser frequency). Furthermore, to simulate a typical nonuniform plasma, the density is assumed to rise linearly from zero at the edge of the plasma and remain linear at least to the critical density point (where plasma frequency equals laser frequency).

$\varepsilon \ll 1$. From (8), $\varepsilon = a_o t_p/R$. R depends on both the density and the critical density (which depends on the laser frequency).

$$R = \frac{n_{ec}}{\nabla n_e} = \left[\frac{4\pi^2 \varepsilon_o m_e c^2}{e^2}\right] \frac{1}{\lambda_o^2 \nabla n_e} \tag{31}$$

The characteristic speed of sound a_o depends in the usual way on the characteristic temperature T_o. T_o becomes, applying (31):

$$T_o = \frac{\lambda_o^4 \nabla n_e (J_o/A)}{\frac{3}{2} k \left[\frac{4\pi^2 \varepsilon_o m_e c^2}{e^2}\right]^2} \tag{32}$$

Then ε can be written

$$\varepsilon = 2.3 \times 10^{-40} (J_o/A)^{1/2} \lambda_o (\nabla n_e)^{3/2} t_p \tag{33}$$

with J_o/A in Joules per square meter, λ_o in microns, ∇n_e in electrons per meter to the fourth, and t_p in seconds.

$\lambda << 1$. Using Spitzer's electron-ion equilibration time with (10)[9]

$$\lambda = 2.3 \times 10^{65} t_p/(J_o/A)^{3/2}(\nabla n_e)^{3/2}\lambda_o^{8} \qquad (34)$$

$\eta_{e,i} << 1$. Only the electron thermal conductivity is considered since it is much larger than ion thermal conductivity. Using Spitzer's thermal conduction coefficient, (11) becomes[9]

$$\eta_e = 1.4 \times 10^{-147}\ t_p(J_o/A)^{5/2}\ \lambda_o^{16}(\nabla n_e)^{9/2} \qquad (35)$$

Then the pulse time t_p, laser energy per unit area J_o/A, laser wavelength λ_o, and electron density gradient ∇n_e arise as the key parameters of the problem. With a particular laser pulse length t_p and wavelength λ_o the various regimes can be shown graphically as a function of J_o/A and ∇n_e. Figures 2, 3, 4, and 5 show when ε, λ, and η_e are large or small for the cases $t_p = 10^{-9}$, 10^{-12} sec. and $\lambda_o = 1.06\mu$, 10.6μ.

The shaded regions in Figures 2-5 represent conditions for which ε, η_e, $\lambda < 1$. Now the validity of a stationary frozen nonconducting solution found will not necessarily be valid near the lines bounding the shaded region.

It is not always enough for the parameters to be small. If the non-dimensional temperature is not of order one, or even if the first or second derivatives take on values not of order one, then the solution may be valid where not expected and may be invalid where not expected. For example, if there is a large temperature gradient at some local point, then significant thermal conduction may occur at that point even though the remainder of the plasma is nonconducting. The frozen ion condition can also break down while λ is small. If the electron temperature is much greater than the ion temperature, then $\lambda\theta_e/\theta_i$ (which appears in 14) is not necessarily small even though λ may be small.

Thus, once having solved the stationary frozen non-thermal conducting case, it will be necessary to look at these effects and re-evaluate the regimes in which the solution is valid.

It is of interest at this point to calculate the parameter α. The solution may take on widely differing character for different values of α. From (9)

$$\alpha = 2.8 \times 10^{88}/(J_o/A)^{3/2}(\nabla n_e)^{5/2}\ \lambda_o^{\ 10} \qquad (36)$$

V. ANALYTICAL SOLUTION

A study of the governing equations shows that the nature of the heating depends on the size of the parameter α. α large corresponds to ∇n_e sufficiently small (such that the underdense portion of the plasma is optically thick). The result is a "heating wave" process whereby successive layers are heated until they become nearly transparent, allowing the beam to penetrate to a deeper layer. α small corresponds to ∇n_e sufficiently large that the underdense portion of the plasma is optically thin. Then, the whole underdense region is heated simultaneously. If α is sufficiently small, the laser will have only a very small heating effect on the plasma and would be ineffective in the production of a CTR plasma.

If the initial temperature of the plasma is much less than T_o, then another small parameter can be defined,

$$\theta_e^{o}(y) = \mu\ g(y) \tag{37}$$

where the maximum value of g(y) is chosen to be one and μ is the small parameter. The governing equation is now

$$0 = \frac{2}{5}\frac{\partial}{\partial y}[\theta_e^{5/2} - \mu^{5/2}g^{5/2}] - \frac{2}{5}\frac{1-\frac{1}{2}\zeta}{\zeta(1-\zeta)}\frac{d\zeta}{dy}[\theta_e^{5/2} - \mu^{5/2}g^{5/2}] + \frac{\alpha\zeta^2}{\sqrt{1-\zeta}}[\theta_e - \mu g] \tag{38}$$

If the final electron temperature is in the neighborhood of T_o, then θ_e will be of order one. Hence, during the hotter stages of heating, the terms in μ will be relatively small. If only terms of order one are considered, (38) becomes

$$0 = \theta_e^{3/2}\frac{\partial\theta_e}{\partial y} - \frac{2}{5}\frac{1-\frac{1}{2}\zeta}{\zeta(1-\zeta)}\frac{d\zeta}{dy}\theta_e^{5/2} + \frac{\alpha\zeta^2}{\sqrt{1-\zeta}}\theta_e + 0(\mu) \tag{39}$$

This is designated the "hot equation".

In the initial stages of the heating, θ_e will be near θ_e^{o} which is of order μ. For this case θ_e must be rescaled; $\theta_e = \mu\theta_\mu$ where θ_μ is of order one. Then (38) becomes

$$0 = \frac{2}{5}\frac{\partial}{\partial y}[\theta_\mu{}^{5/2} - g^{5/2}] - \frac{2}{5}\frac{1 - \frac{1}{2}\zeta}{\zeta(1-\zeta)}\frac{d\zeta}{dy}[\theta_\mu{}^{5/2} - g^{5/2}]$$

$$+ \frac{\alpha}{\mu^{3/2}}\frac{\zeta^2}{\sqrt{1-\zeta}}[\theta_\mu - g] \quad (40)$$

This is designated the "cold equation".

Solving the problem for $\theta_e{}^o \ll 1$ involves solving (39) and (40) separately and then appropriately matching the two solutions to the boundary conditions, initial conditions, and to each other. Solving such a problem where different parts of the solution must be matched with each other requires the method of matched asymptotic expansions such as described in detail by Cole.[10]

Solution to the Cold Equations

A significant parameter $\alpha/\mu^{3/2}$ appears in the cold equation (40) which may be large or small depending on the magnitude of α. There are two important cases, $\alpha \gg \mu^{3/2}$ or $\alpha \ll \mu^{3/2}$.

For $\alpha \ll \mu^{3/2}$, (40) becomes

$$0 = \frac{2}{5}\frac{\partial}{\partial y}[\theta_\mu{}^{5/2} - g^{5/2}] - \frac{2}{5}\frac{1 - \frac{1}{2}\zeta}{\zeta(1-\zeta)}\frac{d\zeta}{dy}[\theta_\mu{}^{5/2} - g^{5/2}] + 0(\alpha/\mu^{3/2})$$

Rearranging this equation gives

$$\frac{\partial}{\partial y}\log[\theta_\mu{}^{5/2} - g^{5/2}] = \frac{\partial}{\partial y}\log\frac{\zeta}{\sqrt{1-\zeta}} + 0(\alpha/\mu^{3/2})$$

which is integrated

$$\theta_\mu = [g^{5/2} + \frac{\ell(\tau)\zeta}{\sqrt{1-\zeta}}]^{2/5} + 0(\alpha/\mu^{3/2}) \quad (41)$$

$\ell(\tau)$ is an arbitrary function of τ that arises in the integration with respect to y. $\ell(\tau)$ is found by applying the boundary condition, which is (29) or (30). The result is

$$\frac{d\ell}{d\tau} = \frac{5}{2}\frac{\alpha}{\mu^{5/2}}\frac{d\Phi}{d\tau}$$

which is integrable

$$\ell = \frac{\alpha}{\mu^{5/2}} \left(\frac{5}{2} \Phi + a\right)$$

where "a" is the constant of integration. "a" can be evaluated by applying the initial condition (28). Since $\Phi(0) = 0$, a = 0 also, and with (37), (41) becomes

$$\theta_\mu = \left[g^{5/2} + \frac{5}{2} \frac{\alpha}{\mu^{5/2}} \frac{\zeta\Phi}{\sqrt{1 - \zeta}}\right]^{5/2} + 0(\alpha/\mu^{3/2}) \tag{42}$$

the solution to the cold equation when $\alpha << \mu^{3/2}$.

For $\alpha >> \mu^{3}/_{2}$, (40) becomes

$$0 = \frac{2}{5} \frac{\mu^{3/2}}{\alpha} \frac{\partial}{\partial y} [\theta_\mu{}^{5/2} - g^{5/2}] + \frac{\zeta^2}{\sqrt{1 - \zeta}} [\theta_\mu - g]$$

$$- \frac{2}{5} \frac{\mu^{3/2}}{\alpha} \frac{1 - \frac{1}{2}\zeta}{\zeta(1 - \zeta)} \frac{d\zeta}{dy} [\theta_\mu{}^{5/2} - g^{5/2}] \tag{43}$$

Since the derivative term is multiplied by the small parameter $\mu^{3/2}/\alpha$, the method of singular perturbations must be used for the solution. Thus the solution to (43) is composed of an outer solution Θ_o, where terms of order $\mu^{3/2}/\alpha$ can be neglected, and an inner solution Θ_i, where the temperature changes rapidly in a very short distance.

The outer solution is to order one,

$$\Theta_o = g \; 0(\mu^{3/2}/\alpha) \tag{44}$$

Finding the inner solution requires expanding the scale of y.

$$\overline{y} = \frac{y - y_o}{\mu^{3/2}/\alpha}$$

where $y_o = y_o(\tau)$ gives the location of the "transition layer." The other functions in (43), $g(y)$ and $\zeta(y)$, will be constant to order one in the scale of $\overline{y}$. For example,

$$\zeta(y) = \zeta(y_o + \frac{\mu^{3/2}}{\alpha} \overline{y}) = \zeta(y_o) + \frac{\mu^{3/2}}{\alpha} \overline{y} \left.\frac{d\zeta}{dy}\right|_{y = y_o} + 0(\mu^3/\alpha^2)$$

Then to order one, the inner equation is

$$0 = \Theta_i^{\ 3/2} \frac{\partial \Theta_i}{\partial \bar{y}} + \frac{\zeta^2(y_o)}{\sqrt{1 - \zeta(y_o)}} [\Theta_i - g(y_o)] + 0(\mu^{3/2}/\alpha) \tag{45}$$

which can be integrated

$$\int \frac{\Theta_i^{\ 3/2} d\Theta_i}{\Theta_i - g(y_o)} = m(\tau) - \frac{\zeta^2(y_o)}{\sqrt{1 - \zeta(y_o)}} \bar{y} + 0(\mu^{3/2}/\alpha) \tag{46}$$

$m(\tau)$ is the function of time that arises in the integration with respect to $\bar{y}$.

The integral on the left side of (46) can be evaluated using ordinary methods of integration. Conducting the integration, (46) becomes

$$\frac{2}{3}\left(\frac{\Theta_i}{g}\right)^{3/2} + 2\left(\frac{\Theta_i}{g}\right)^{1/2} + \log \frac{\left(\frac{\Theta_i}{g}\right)^{1/2} - 1}{\left(\frac{\Theta_i}{g}\right)^{1/2} + 1} \tag{47}$$

$$= \frac{1}{[g(y_o)]^{3/2}} [m(\tau) - \frac{\zeta^2(y_o)}{\sqrt{1 - \zeta}} \bar{y}] + (\mu^{3/2}/\alpha)$$

The complicated expression on the left side (47) can be expanded for limiting values of Θ_i/g.

For Θ_i/g near one, (47) can be solved as an expansion for Θ_i/g,

$$\Theta_i = g \{1 + 4 \exp \frac{1}{[g(y_o)]^{3/2}} [- \frac{8}{3} + m(\tau) - \frac{\zeta^2(y_o)}{\sqrt{1 - \zeta(y_o)}} \bar{y}]$$

$$+ 0(\mu^{3/2}/\alpha)$$

which is valid for $\Theta_i/g - 1 << 1$.

As $\overline{y}$ goes to positive infinity, Θ_i is seen to reduce exponentially fast to g which is just the outer solution (44). Thus the inner solution on the positive $\overline{y}$ side of the transition layer.

For Θ_i/g much larger than one, (47) can be again solved as an expansion,

$$\Theta_i = [\frac{3}{2} m(\tau) - \frac{3}{2} \frac{{}^2(yo)}{\sqrt{1 - \zeta(y_o)}} \overline{y}]^{2/3} + O(\mu^{3/2}/\alpha) \tag{49}$$

which is valid for $\Theta_i/g >> 1$. Θ_i is seen to grow very large if $\overline{y}$ gets large negative. Hence, for large negative $\overline{y}$, (49) must be made to match the solution to the hot equation.

Solution to the Hot Equation

When the energy in the laser pulse is large compared to the initial internal energy of the plasma, then generally $(\theta_e)_{final}/\theta_e^{\circ} >> 1$. In this case $(\theta_e)_{final}$ is essentially independent of $\theta_e^{\circ}(y)$. The "hot equation" (39) governs this case. Dividing (39) by θ_e gives a linear first order equation in $\theta_e^{3/2}$,

$$\frac{\partial\theta_e^{3/2}}{\partial y} - \frac{3}{5} \frac{1 - \frac{1}{2}\zeta}{\zeta(1 - \zeta)} \frac{d\zeta}{dy} \theta_e^{3/2} = - \frac{3}{2} \frac{\alpha\zeta^2}{\sqrt{1 - \zeta}} + O(\mu)$$

which can easily be solved to give

$$\theta_e = \frac{\zeta^{2/5}}{(1 - \zeta)^{1/5}} \{c(\tau) - \frac{3}{2} \alpha \int_o^y \frac{\zeta^{7/5} dy}{(1 - \zeta)^{1/5}} \}^{2/3} + O(\mu)$$

The function $c(\tau)$ that arises in the integration with respect to y is determined by applying the boundary condition (29) or (30),

$$\frac{d}{d\tau} [c(\tau)]^{5/3} = \frac{5}{2} \alpha \frac{d\Phi}{d\tau}$$

or

$$c(\tau) = \alpha^{3/5} (\frac{5}{2} \Phi + b)^{3/5}$$

b is the constant of integration and will be determined by the matching with the cold solution. Hence,

$$\theta_e = \frac{\alpha^{2/5}\zeta^{2/5}}{(1-\zeta)^{1/5}} \left\{ \left(\frac{5}{2}\Phi + b\right)^{3/5} - \frac{3}{2}\alpha^{2/5}\int_o^y \frac{\zeta^{7/5}\,dy}{(1-\zeta)^{1/5}} \right\}^{2/3} + 0(\mu) \tag{50}$$

The expression (50) is only physically realistic when the term in braces is greater than or equal to zero. It is seen that given a τ, this term is positive for y less than a certain value.

The physical impossibility of its being negative is seen if the resulting expression for $i(y,\tau)$ is written using (24).

$$i = \left\{ \left(\frac{5}{2}\Phi + b\right)^{3/5} - \frac{3}{2}\alpha^{2/5}\int_o^y \frac{\zeta^{7/5}\,dy}{(1-\zeta)^{1/5}} \right\}^{5/3} \frac{\frac{d\Phi}{d\tau}}{\left(\frac{5}{2}\Phi\right)^{2/5}}$$

The term in braces becoming negative corresponds to the intensity becoming negative which is impossible. The hot solution is only valid for positive values of the term in braces. The cold solution (45) must be used elsewhere.

If $\alpha^{2/5} << 1$, (50) can be expanded with $\alpha^{2/5}$ as the small parameter

$$\theta_e = \frac{\alpha^{2/5}\,\zeta^{2/5}}{(1-\zeta)^{1/5}} \left(\frac{5}{2}\Phi + b\right)^{2/5} + 0(\alpha^{4/5},\mu) \tag{51}$$

Matching Hot and Cold Solutions

In the various expressions that compose the solution (42), (44), (47), and (50), various unknowns arose: b, $m(\tau)$ and $y_0(\tau)$. These unknowns will be determined by matching the different components with each other for the possible ranges of the parameter α.

$\alpha << \mu^{5/2}$

The cold solution is given by (42). The first term in brackets is of order one and the second term is of order $\alpha/\mu^{5/2}$ and is relatively small. Then (42) can be expanded about $g(y)$ (where g is nonvanishing) giving

$$\theta_\mu = g + \frac{\alpha}{\mu^{5/2}} \frac{\zeta\Phi}{g^{3/2}\sqrt{1-\zeta}} + 0(\alpha/\mu^{3/2}) \tag{52}$$

It is immediately seen that at the end of the heating ($\Phi = 1$), the term in $\alpha/\mu^{5/2}$ is still small compared to g and thus there is no need to seek a hot regime. The temperature θ_e remains in the cold regime throughout the laser pulse and is governed by (52).

Physically this means that very little of the laser energy is actually absorbed by the plasma. The whole underdense region is penetrated by the heating so that a reflected light wave will arise. Since the intensity of the reflected light will be imperceptibly diminished due to weak absorption, its effect will be to roughly double the heating due to the incident light, i.e., the term in $\alpha/\mu^{5/2}$ is doubled.

Neglecting the effect of the reflected wave, the temperature produced is

$$\theta_e = \theta_e{}^\circ(y) + \frac{\alpha}{[\theta_e{}^\circ(y)]^{3/2}} \frac{\zeta\Phi}{\sqrt{1-\zeta}} + O(\alpha/\mu^{1/2}) \tag{53}$$

This is shown schematically in Figure 6 for a linear density gradient, a constant initial temperature, and for $\alpha/\mu^{5/2} = 1/10$.

$$\mu^{5/2} \ll \alpha \ll \mu^{3/2}$$

The cold solution is again given by (42), but now the term in $\alpha/\mu^{5/2}$ dominates the term of order one. Expanding about the term in α (where Φ, ζ are non-vanishing) gives

$$\theta_\mu = \frac{\alpha^{2/5}}{\mu} \frac{\zeta^{2/5}(\frac{5}{2}\Phi)^{2/5}}{(1-\zeta)^{1/5}} + O(1) \tag{54}$$

This must match with the small α limit of the hot solution (51). The matching requires that the constant b in (51) be zero. Once again the entire underdense region is heated and a reflected light wave will arise whose intensity will be diminished little since α is small. The effect of the reflected wave then is to double the temperature in (54).

Neglecting the effect of the reflected wave, a composite solution can be constructed which possesses the features of the hot solution (51) and cold solution (42),

$$\theta_e = [\theta_e{}^{\circ 5/2} + \frac{5}{2} \frac{\alpha\zeta\Phi}{\sqrt{1-\zeta}}]^{2/5} + O(\alpha^{2/5}) \tag{55}$$

$$\mu^{3/2} \ll \alpha$$

The cold solution is given by an outer solution (44) and a transition layer solution (47). As was seen before, the transition layer [located at $y_0(\tau)$] was matched with the outer solution for $y > y_0(\tau)$. Now, (47) must be matched with the hot solution (50). The asymptotic limit for Θ_i large is given by (49). The limit of θ_e small in the hot solution must be calculated. This matching problem is demonstrated schematically in Figure 7. θ_e in (50) achieves small values near the point where the term in braces vanishes. Given τ, the term in braces vanishes at some $y_w = y_w(\tau)$. Expand the scale of y in the neighborhood of y_w ,

$$y_\nu = \frac{y - y_w}{\nu}$$

where ν is a small parameter to be determined in the matching. Then (50) becomes

$$\theta_e = [\frac{3}{2}\alpha\nu \frac{\zeta^2(y_w)}{\sqrt{1-\zeta(y_w)}} \quad (-y_\nu)\ ]^{2/3} + 0(\mu)$$

This can be matched to (49) if $\nu = \mu^{3/2}/\alpha$, $m(\tau) = 0$, and $y_0(\tau) = y_w(\tau)$, in which case $y_\nu = \bar{y}$.

A composite solution valid over all y cannot be written in this case but the components can be summarized,

$$\theta_e = \frac{\alpha^{2/5}\zeta^{2/5}}{(1-\zeta)^{1/5}} \{(\frac{5}{2}\Phi)^{3/5} - \frac{3}{2}\alpha^{2/5}\int_0^y \frac{\zeta^{7/5}\,dy}{(1-\zeta)^{1/5}}\}^{2/3} + 0(\mu)$$

for $y < y_w(\tau)$

$$\theta_e = \theta_e{}^\circ + 0(\mu^{5/2}/\alpha) \quad \text{for } y > y_w(\tau)$$

$$[\theta_e{}^\circ(y_w)]^{3/2}\{\frac{2}{3}[\frac{\theta_e}{\theta_e{}^\circ(y_w)}]^{3/2} + 2[\frac{\theta_e}{\theta_e{}^\circ(y_w)}]^{1/2}$$

$$+ \log\frac{[\frac{\theta_e}{\theta_e{}^\circ(y_w)}]^{1/2} - 1}{[\frac{\theta_e}{\theta_e{}^\circ(y_w)}]^{1/2} + 1} = -\frac{\alpha\zeta^2(y_w)}{\sqrt{1-\zeta(y_w)}}[y - y_w] + 0(\mu^3/\alpha) \tag{56}$$

for $y - y_w = 0(\mu^{3/2}/\alpha)$

A "heating wave" is seen to arise which separates hot parts of the plasma from cold parts. The location of the "wave," $y_w(\tau)$ is given implicitly by

$$(\frac{5}{2}\Phi)^{3/5} = \frac{3}{2}\alpha^{2/5}\int_0^{y_w(\tau)} \frac{\zeta^{7/5}dy}{(1-\zeta)^{1/5}} \tag{57}$$

As Φ increases from zero, y_W is seen to increase monotonically with Φ .

If α is very small, the wave travels almost infinitely fast and reaches the critical density very early in the laser pulse. From that time on, the laser heats the entire underdense portion of the plasma simultaneously and the temperature profile produced is

$$\theta_e = (\frac{5}{2}\frac{\alpha\zeta\Phi}{\sqrt{1-\zeta}})^{2/5} + 0(\alpha^{4/5},\mu) \tag{58}$$

A "hot spot" is generated near $\zeta = 1$, the critical density. Of course the original expression for absorption coefficient (17) breaks down very near $\zeta = 1$ and remains finite at the critical density. Nevertheless, a hot spot is produced in this region. This limit can be labeled the "simultaneous heating case." In this case the actual temperature produced will be roughly twice that given in (50) since the light wave is reflected at the critical density and travels backwards through the region with very little intensity attenuation [see (16)]. Hence, most of the laser radiation is lost to reflection.

If α is larger than a certain value, the wave travels much more slowly and does not even reach the critical density by the end of the laser pulse. Hence, in this case there is no isolated hot spot as for α small. This limit can be termed the "heating wave case" since a heating wave continues to move into the plasma throughout the laser pulse. The heating is by no means "simultaneous." In this case all of the laser energy is absorbed.

In summary, stationary-frozen-nonconducting heating of a cool plasma will occur in one of four regimes. If $\alpha \ll \mu^{5/2}$, then the plasma is only slightly heated and the heating is simultaneous. A weak hot spot occurs at the critical density and nearly all the laser energy is lost to reflection. If $\mu^{5/2} \ll \alpha \ll \mu^{3/2}$, the plasma is heated to temperatures much hotter than the initial temperature but still most of the laser energy is lost to reflection. The heating is simultaneous and a stronger hot spot appears. If $\mu^{3/2} \ll \alpha \ll 1$, a very rapid heating wave traverses the underdense region early in the pulse. After that the heating is simultaneous and a significant hot spot appears at the critical density. During the simultaneous heating stage, a significant part of the laser energy is lost to reflection but not as much as in the previous cases. If $\alpha \gg 1$, a much slower heating wave propagates into the plasma and the wave does not penetrate all the way to the critical density. No hot spot or reflected wave appears, and all of the laser energy is absorbed.

VI. APPLICATION TO A LINEAR DENSITY GRADIENT

The solutions given in the last section can be easily calculated for a linear density gradient, $\zeta = y$. These results for three different values of the parameter α are shown in Figures 8, 9, 10.

Figure 8 clearly demonstrates the heating wave moving into the plasma In this case ($\alpha = 1000$) the heating wave only gets about halfway to the critical density by the end of the laser pulse. In Figure 9 ($\alpha = 4$) the heating wave reaches the critical density just before the end of the laser pulse, and a weak hot spot is generated at the critical density. In Figure 10 ($\alpha << 1$) the heating wave travels almost infinitely fast in traversing the underdense region and hence is not shown. A strong hot spot arises at the critical density.

While Figures 8, 9, 10 give the analytical features of the solutions, they are given in terms of dimensionless quantities and do not show clearly the temperatures and other conditions that are attained. Figures 11, 12 are summary plots for laser wavelengths $\lambda_o = 1.06\mu$ and 10.6μ respectively. The "maximum temperatures" given are the maximum temperatures without the hot spot. Naturally, when a hot spot appears, there will be a much hotter temperature in a very localized region. The "plasma length" given is the thickness of the plasma that is irradiated by the laser. This thickness will be R for the simultaneous heating case, and will be somewhat less for the heating case (since the wave does not traverse the whole underdense region).

It is noted that the 10.6μ laser heats in the simultaneous heating regime for much smaller electron density gradients than for the 1.06μ laser, i.e., a relatively long plasma may be in the simultaneous heating regime under 10.6μ radiation but in the heating wave regime under 1.06μ radiation. It is also seen by comparing Figures 11 and 12 that the temperatures achieved are independent of the laser wavelength in the simultaneous heating regime. However, the longer wavelength heating produces higher temperatures in the wave heating regime.

It has been previously noted that not all the radiation is absorbed in the simultaneous heating case but that much is lost to reflection. It is important to know what fraction of energy is absorbed -- both from the standpoint of efficiency and from the fact that reflected radiation may damage the laser. Integrating the temperature for $\alpha << 1$ (58), over the thickness of the underdense region, the fraction of energy absorbed can be calculated. This fraction is doubled to take into account the absorption of reflected light, and the result is

$$\frac{J_{abs}}{J} = 2\left(\frac{5}{2}\alpha\right)^{2/5} \int_0^1 \frac{\zeta^{7/5}\,dy}{(1-\zeta)^{1/5}} \tag{59}$$

which for a linear density gradient reduces to

$$\frac{J_{abs}}{J_o} = 1.72\ \alpha^{2/5} \tag{60}$$

These results are shown in Figure 13.

VII. CONCLUSIONS

Analytic solutions have been developed for the laser heating of a one-dimensional plasma under the assumption that the plasma is stationary, non-thermally conducting, and that only the electrons are heated (the ions being frozen at their initial temperature). These assumptions were found to be reasonable for many cases of interest. In these cases, the laser pulse is sufficiently short that the processes of plasma motion, thermal conduction and electron-ion equilibration don't have time to make a significant effect. These assumptions reduce the problem (aside from equations of state) to a pair of equations; a radiative transfer equation and an equation for the temperature containing an energy source term. These equations were combined and solved for all ranges of remaining parameter α by the method of matched asymptotic expansions.

The use of a one-dimensional plasma was motivated by the need to make the mathematics tractable. No real plasma is strictly one-dimensional but many plasma geometries of interest are nearly one-dimensional and the analytic results of this paper can be applied to such plasmas with reasonable accuracy. An example of thermonuclear interest is the longitudinal heating of a column of plasma that has a uniform density across the column, such as might be found in a plasma focus or a theta pinch. Another example is the radial heating of such a column which is quasi-one-dimensional if the absorption length $1/\kappa_o$ is less than the radius of the column. Such a plasma with a number density of $10^{19}/cm^3$ heated by a longitudinally aimed laser of wavelength 10.6μ to a temperature of 10 Kev could be contained with a magnetic field of 3 megagauss (as might be developed in a plasma focus). For plasmas that are not nearly one-dimensional, the results of this study are still valuable in understanding qualitative results.

The analytic solution shows markedly different behavior for different ranges of α. For thick plasmas (corresponding to α large)the response is characterized by a heating wave proceeding into the plasma. Behind the wave is a hot nearly transparent plasma being irradiated by the laser. Ahead of the wave is an unheated opaque plasma. Physically, the motion of the wave corresponds to the heating of successive layers of opaque plasma to temperatures at which they become transparent and allow the radiation to pass on to deeper layers. For α greater than a certain distinct value (depending on the density profile but always on the order of one), the heating wave fails to penetrate to the critical density by the end of the laser pulse. Then none of the radiation is reflected back to the critical density.

For smaller α, the heating wave reaches the critical density before the laser pulse ends. At that time, reflection occurs at the critical density and some of the light is reflected. Although the heating is simultaneous once the heating wave traverses the underdense region, it is by no means uniform -- an extreme hot spot is created at the critical density point. For relatively thin plasmas (α somewhat less than one) the heating wave traverses the underdense region extremely fast and all but the earliest stages of the heating are "simultaneous."

For α smaller than the small parameter $\mu^{3/2}$, the entire heating process is simultaneous with no heating wave at all. The parameter μ is a measure of the initial temperature. The efficiency of the heating drops further as more of the laser energy is lost to reflection. For α even smaller (less than $\mu^{5/2}$), the heating is so ineffective that the temperature is scarcely changed at all.

It is clear that in the preparation of the plasma to be heated, the temperatures (or other conditions sought) can be optimized by proper choice of the parameter α. For a given laser energy per unit area and wavelength, α depends on the density gradient. Hence, proper tailoring of the density gradient beforehand will give the best results, whether that be maximum temperature or maximum yield from a thermonuclear reaction.

The solution found in this paper is only for a stationary, non-thermally conducting, frozen ion plasma. But the solutions for a stationary conducting plasma, and a stationary equilibrium plasma (electron and ion temperatures the same) are closely related. In the case of a conducting plasma, the effect of conduction will be simply to smooth out the nonuniformities in temperature that are produced. In the case of a plasma with electron-ion equilibrium, the effect of the equilibration will be to divide equally the energy between the electrons and ions. In this case, the result is almost identical to the frozen ion solution presented in this paper. These two problems are interesting extensions of the work in this paper and form topics for further study.

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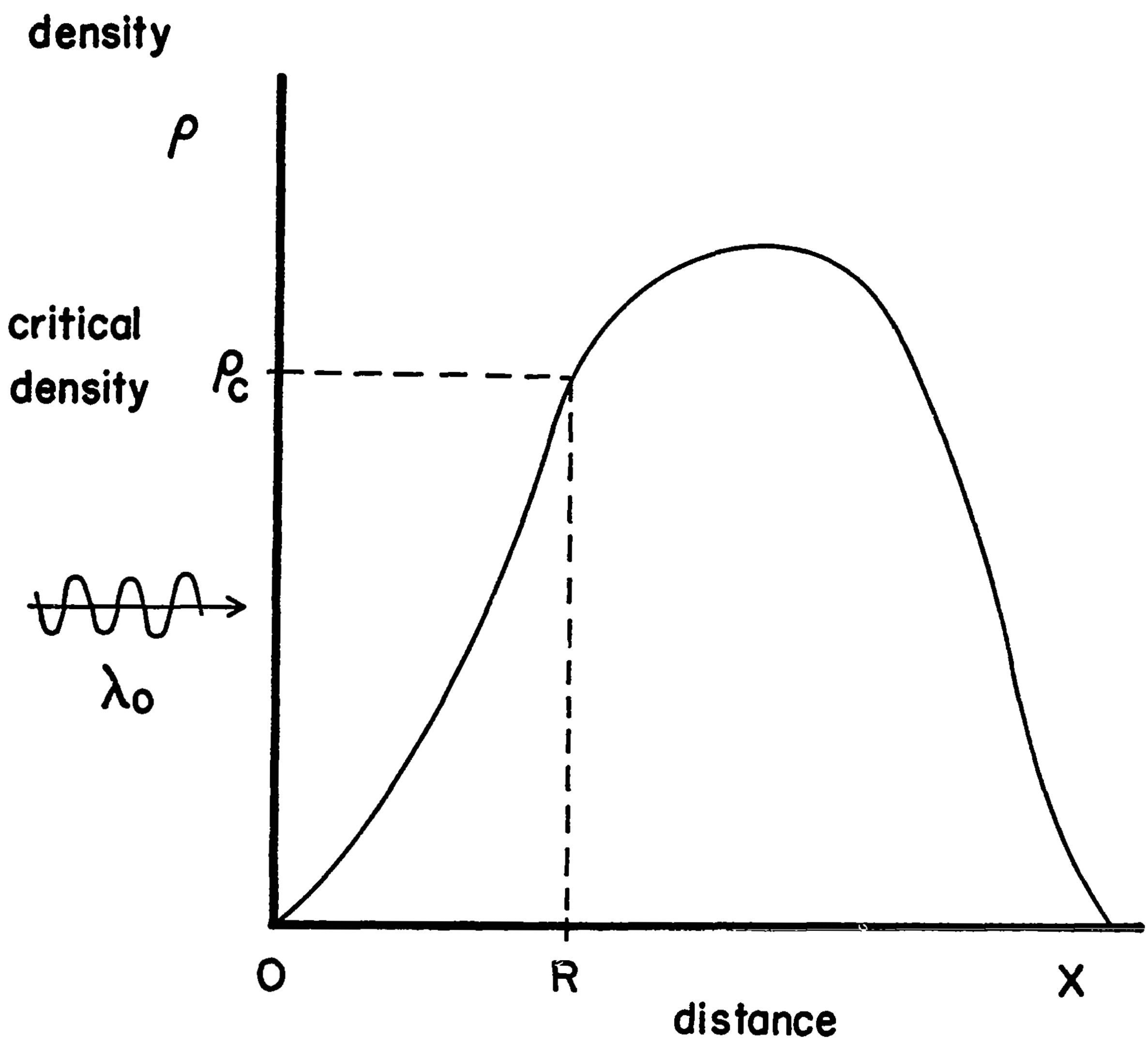

Figure 1. Plasma Configuration

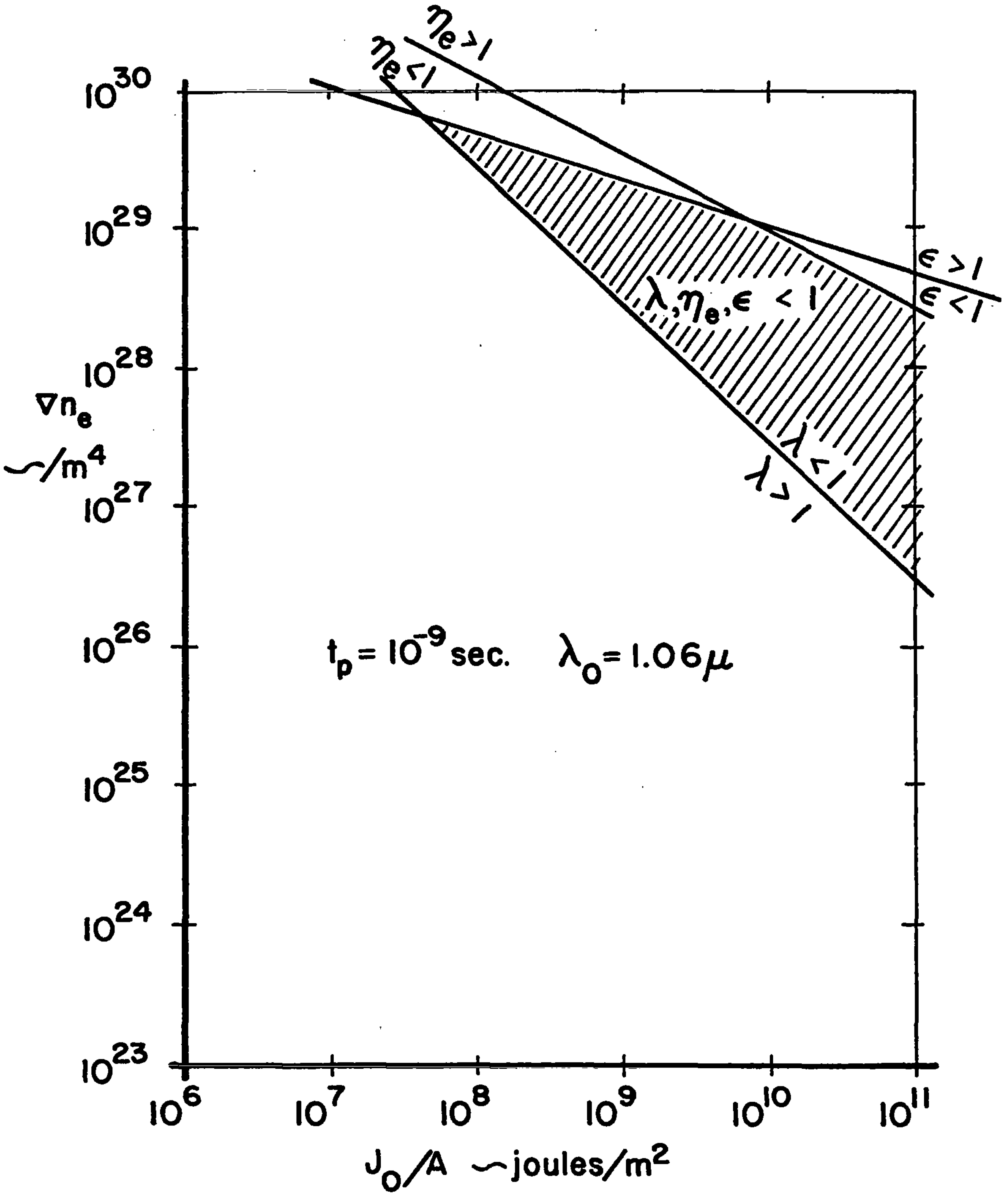

Figure 2. Region of Validity of the Stationary, Nonconducting Frozen Limit for a Nanosecond Pulse from a Nd^+ Glass Laser

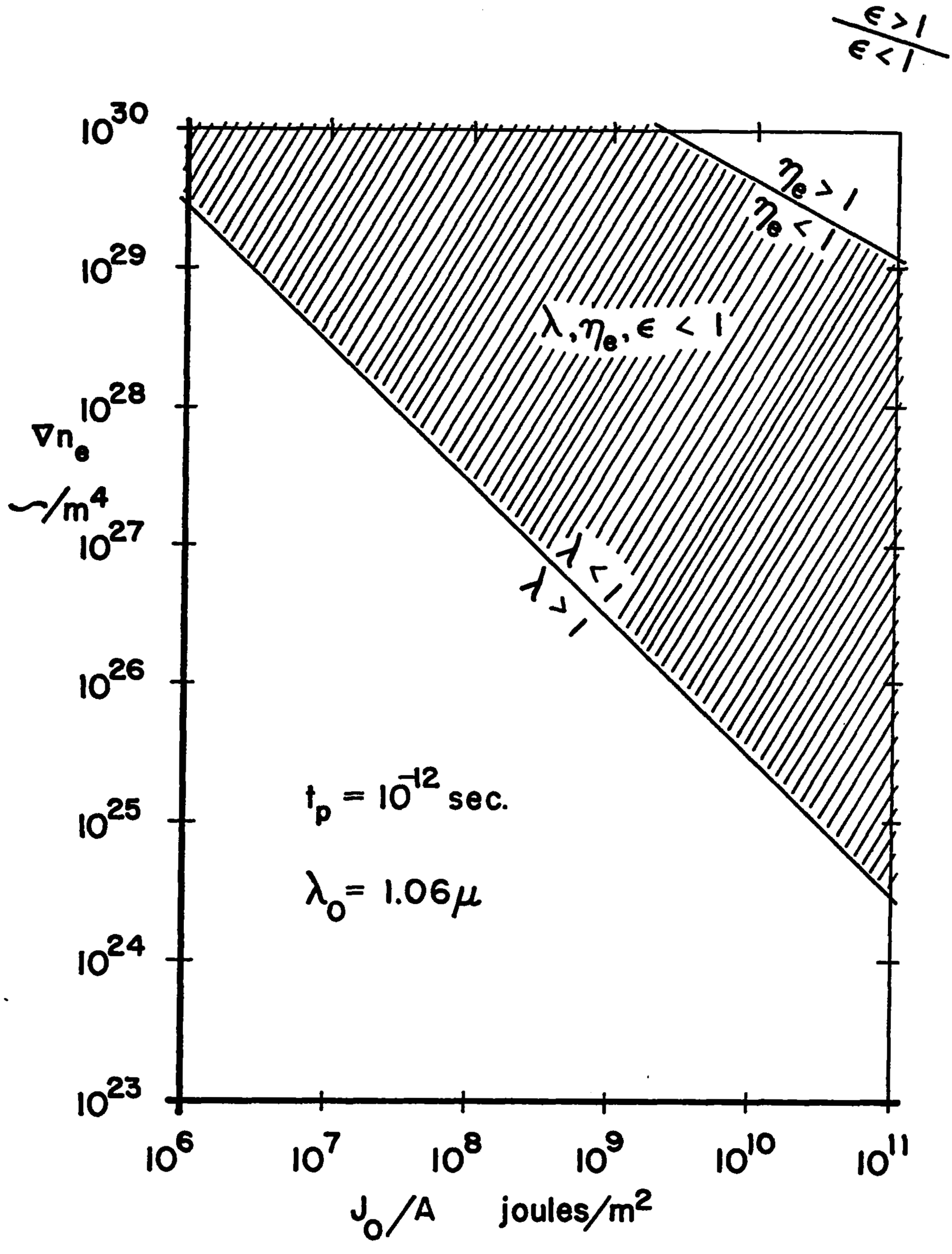

Figure 3. Region of Validity of the Stationary, Nonconducting Frozen Limit for a Picosecond Pulse from a Nd^+ Glass Laser

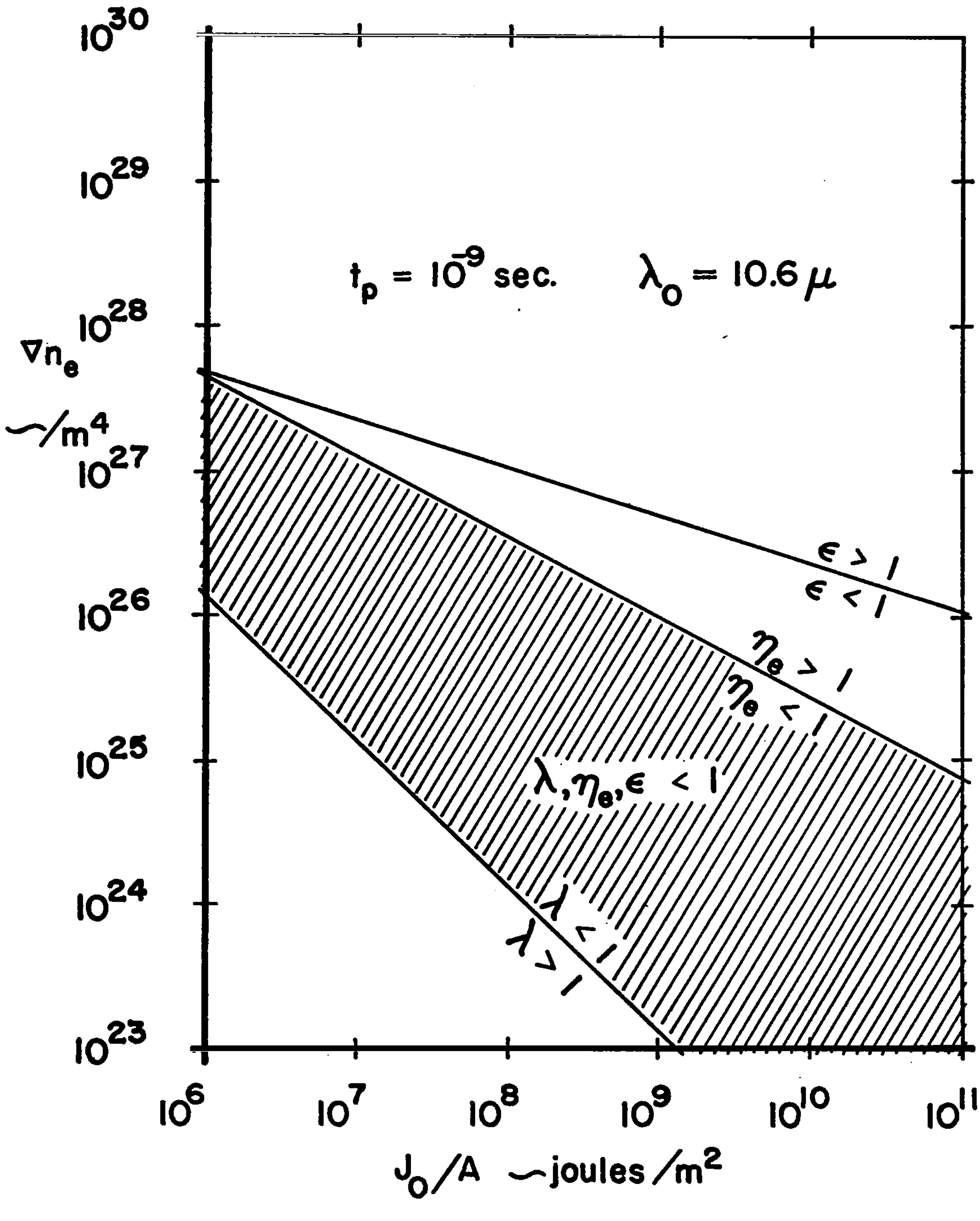

Figure 4. Region of Validity of the Stationary, Nonconducting Frozen Limit for a Nanosecond Pulse from a CO_2 Gas Laser

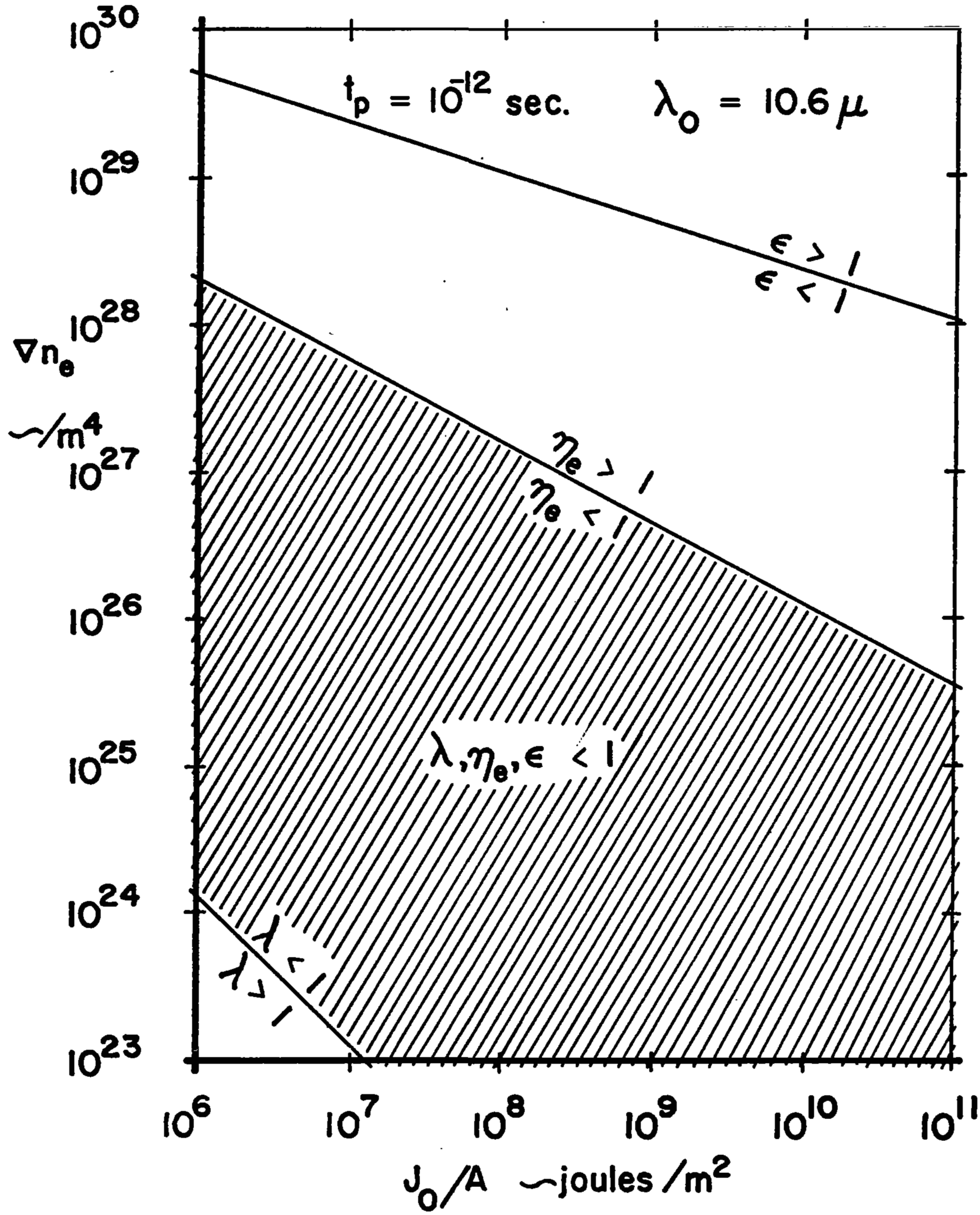

Figure 5. Region of Validity of the Stationary, Nonconducting Frozen Limit for a Picosecond Pulse from a CO_2 Gas Laser

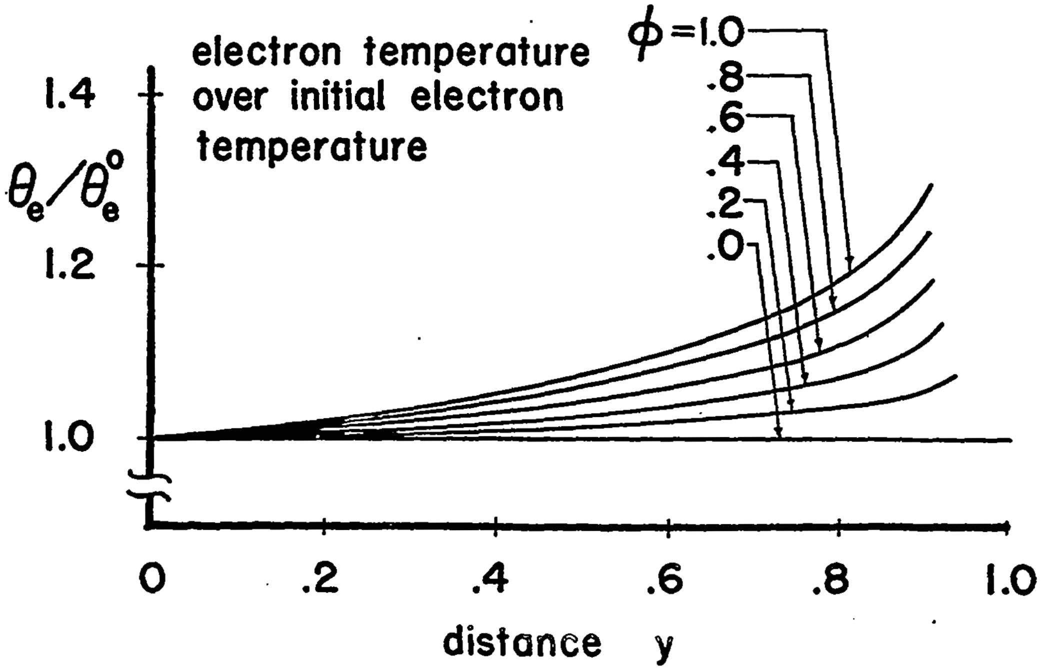

Figure 6. Temperature Distribution Produced when $\alpha/\mu^{5/2} = 0.10$

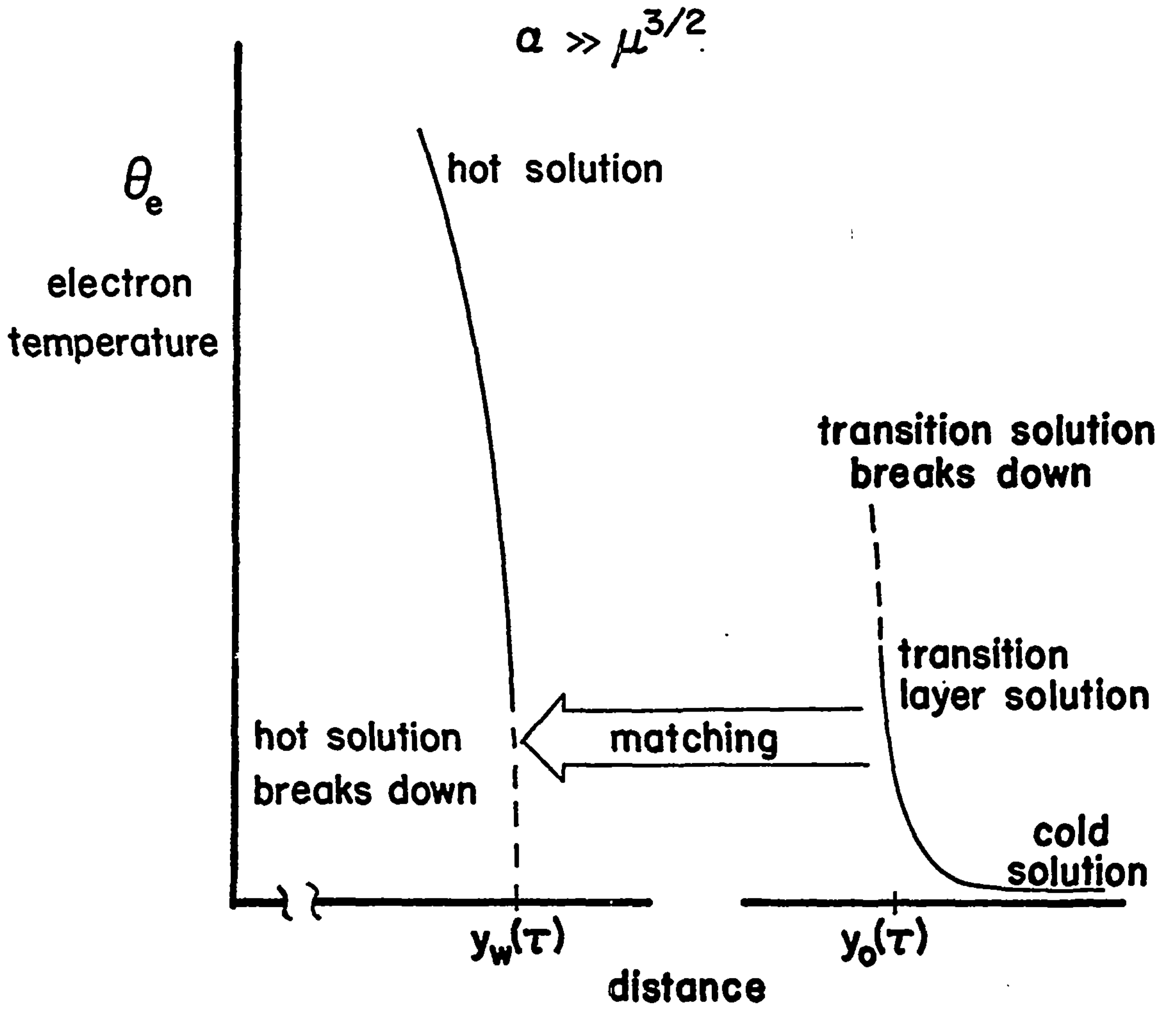

Figure 7. Matching of the Hot Solution to the Transition Layer and the Cold Solution

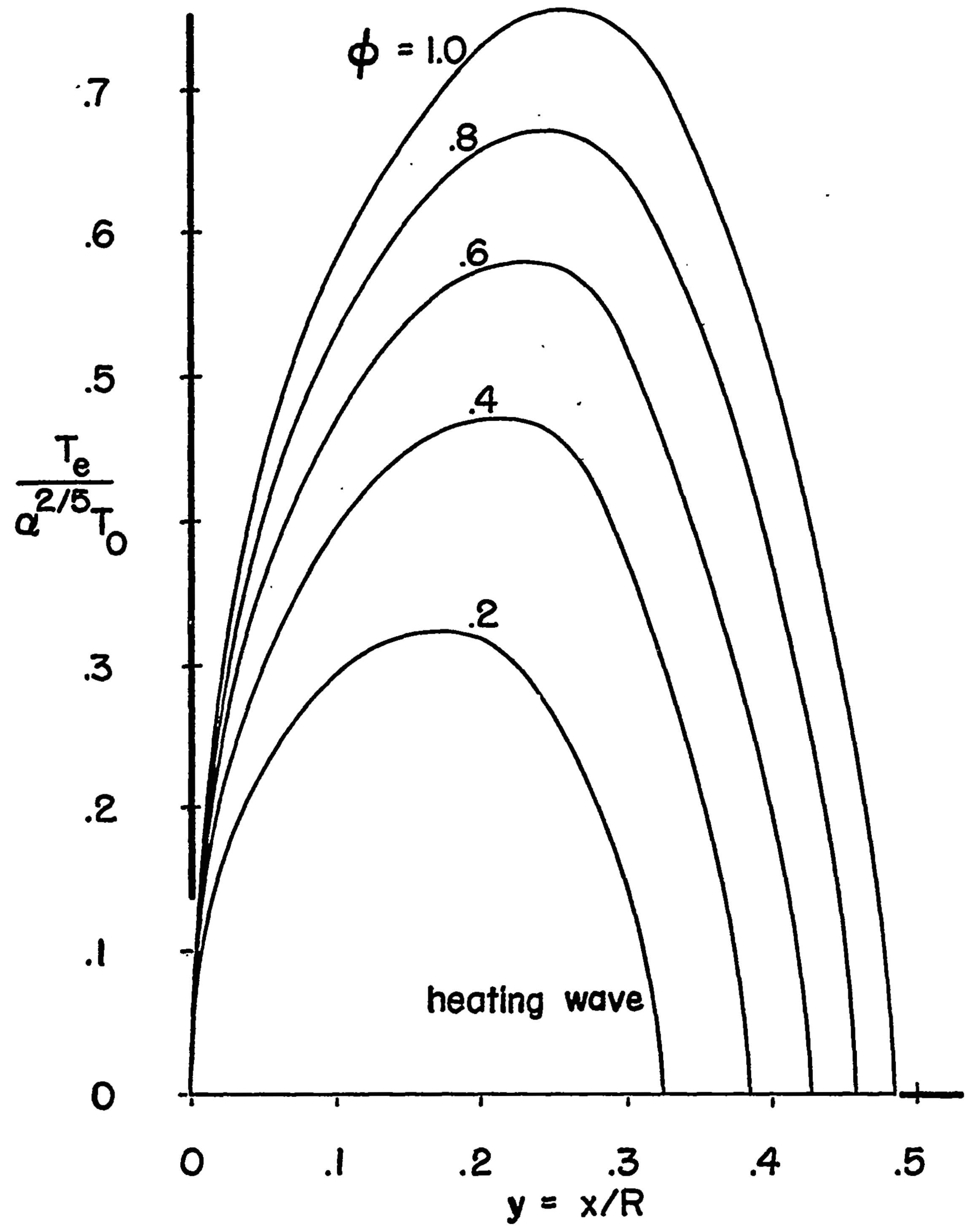

Figure 8. Temperature Distribution for a Linear Density Gradient with α = 1000

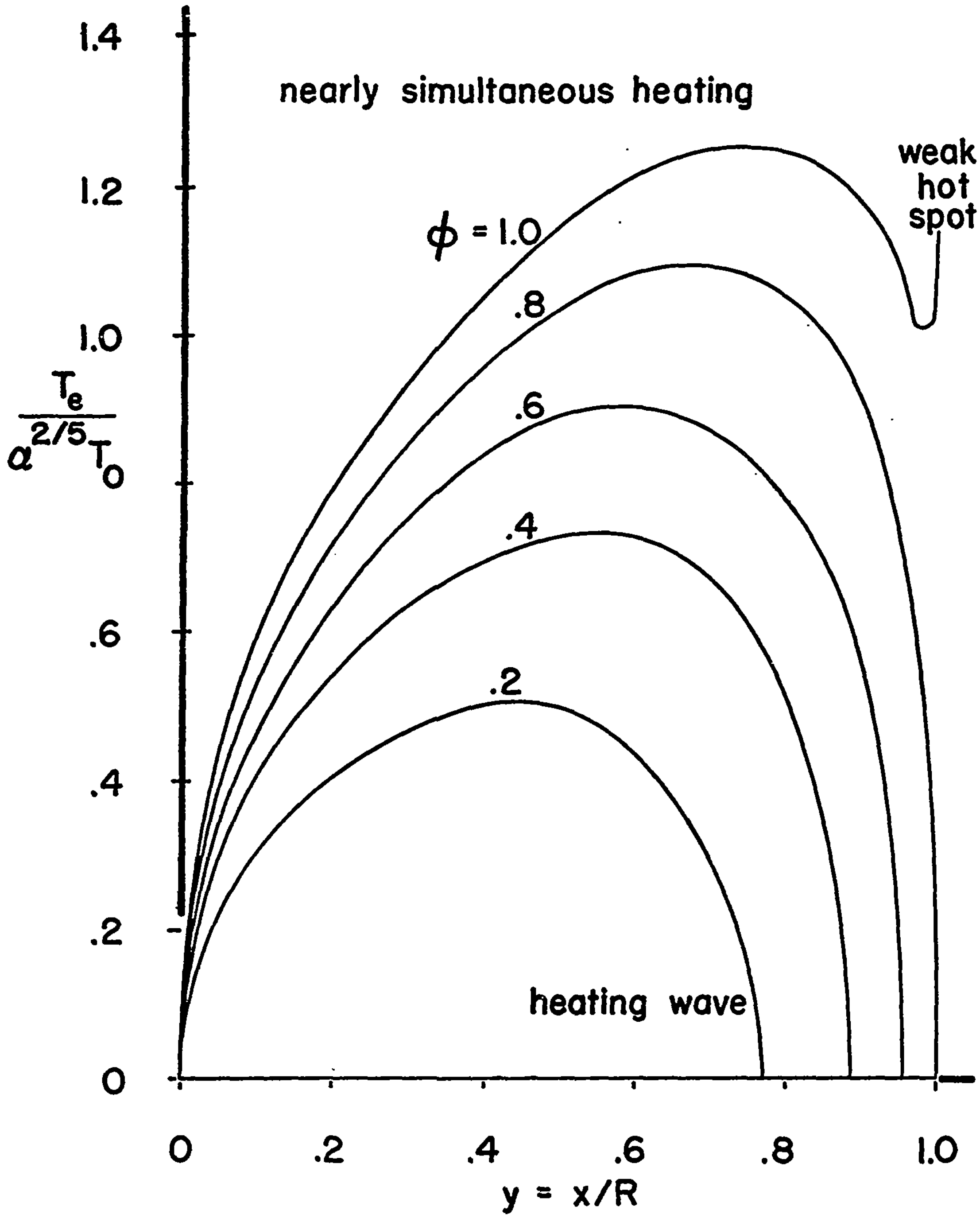

Figure 9, Temperature Distribution for a Linear Density Gradient with $\alpha = 4$

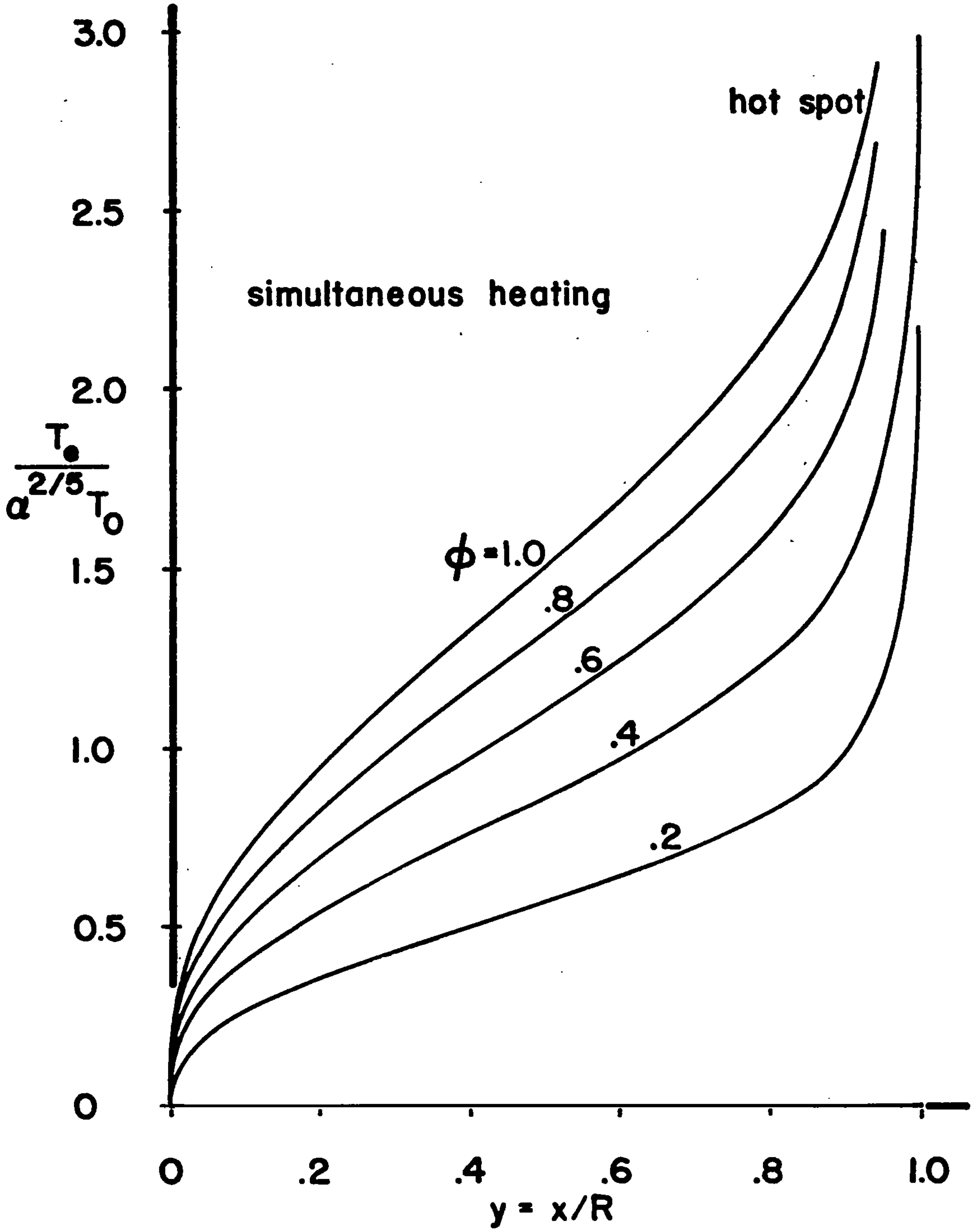

Figure 10. Temperature Distribution for a Linear Density Gradient with $\mu^{5/2} << \alpha << 1$

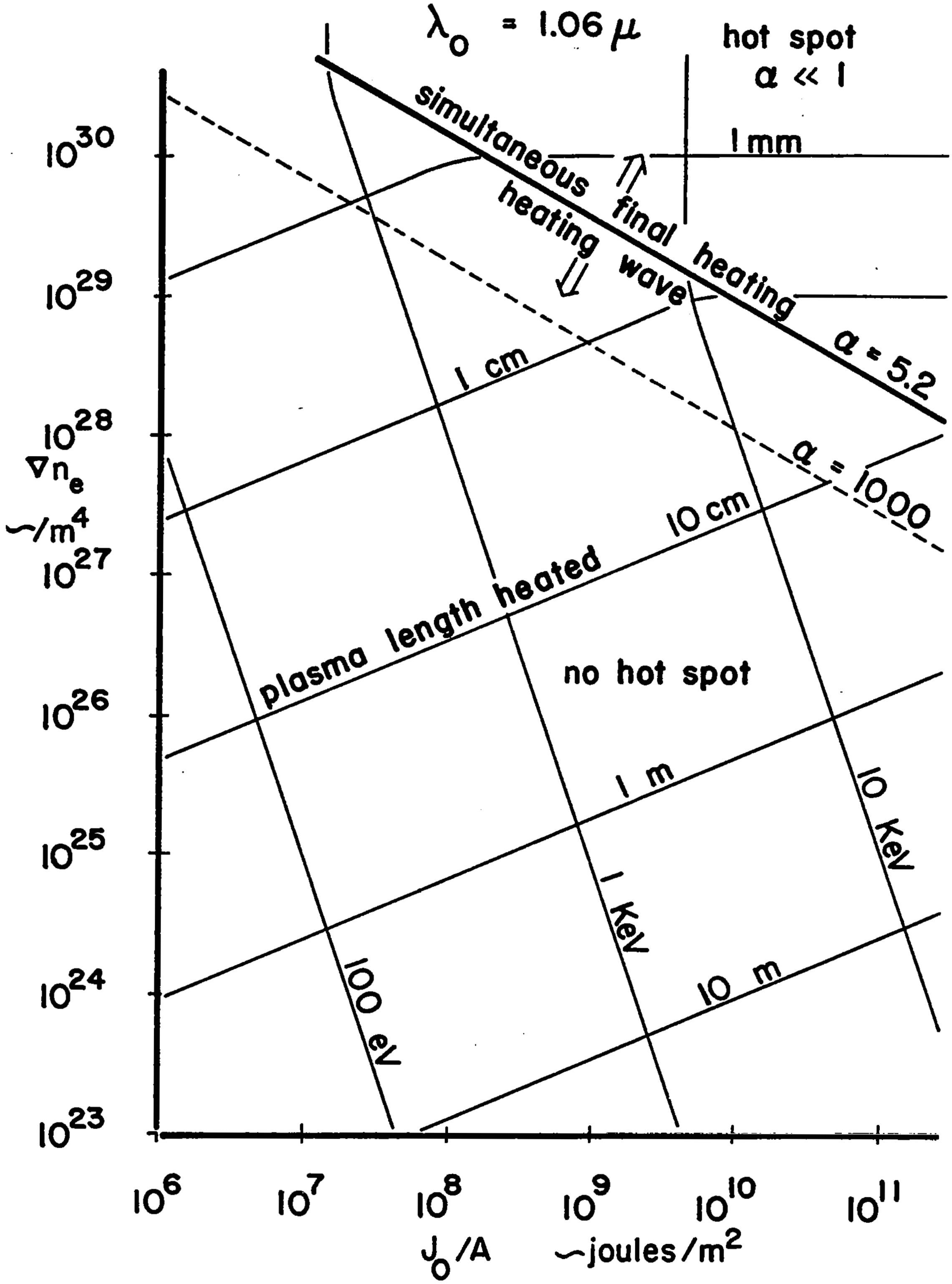

Figure 11. Summary of Plasma Conditions which can be Achieved for a Linear Density Gradient with a Nd^+ Glass Laser

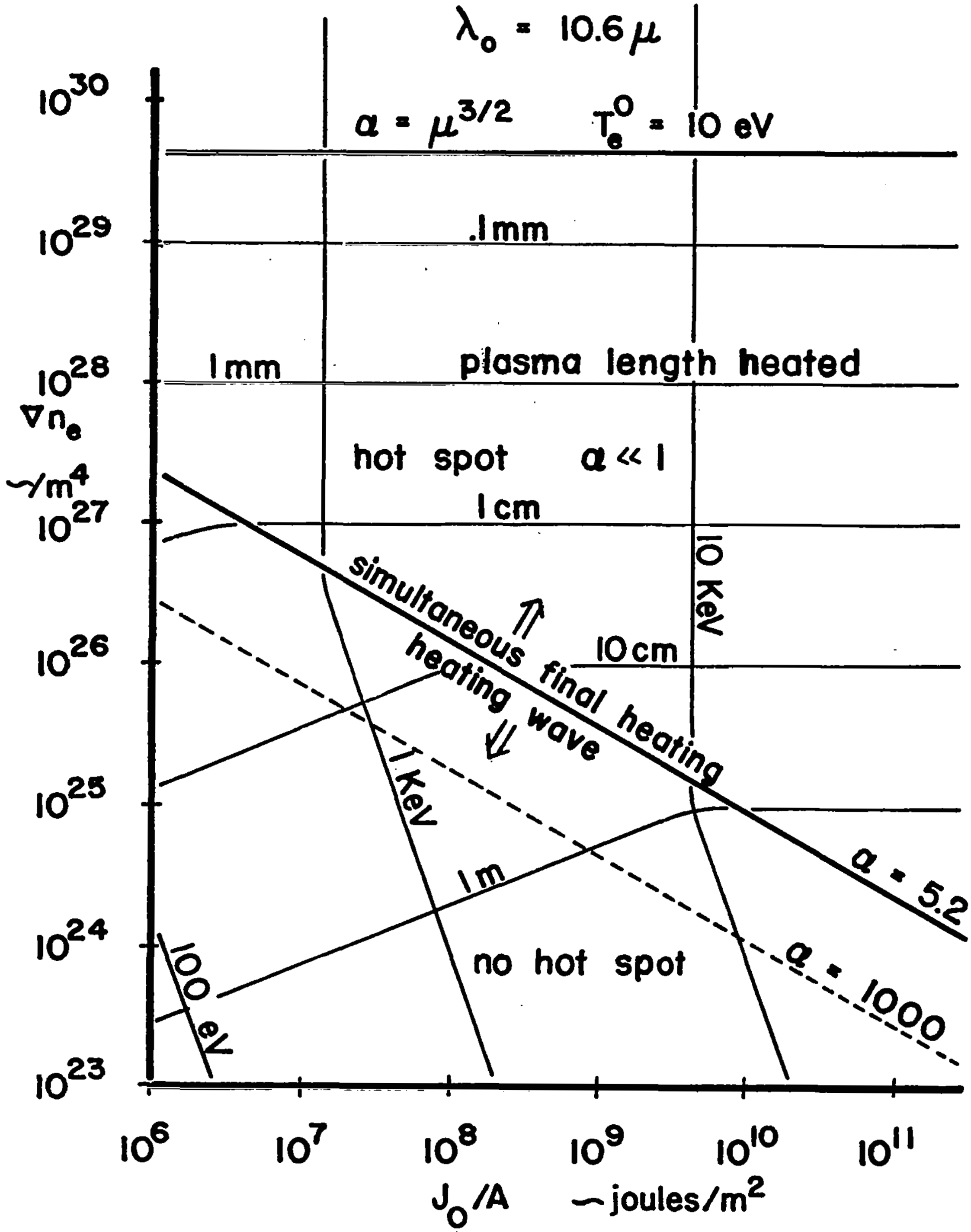

Figure 12. Summary of Plasma Conditions which can be Achieved for a Linear Density Gradient with a CO_2 Gas Laser

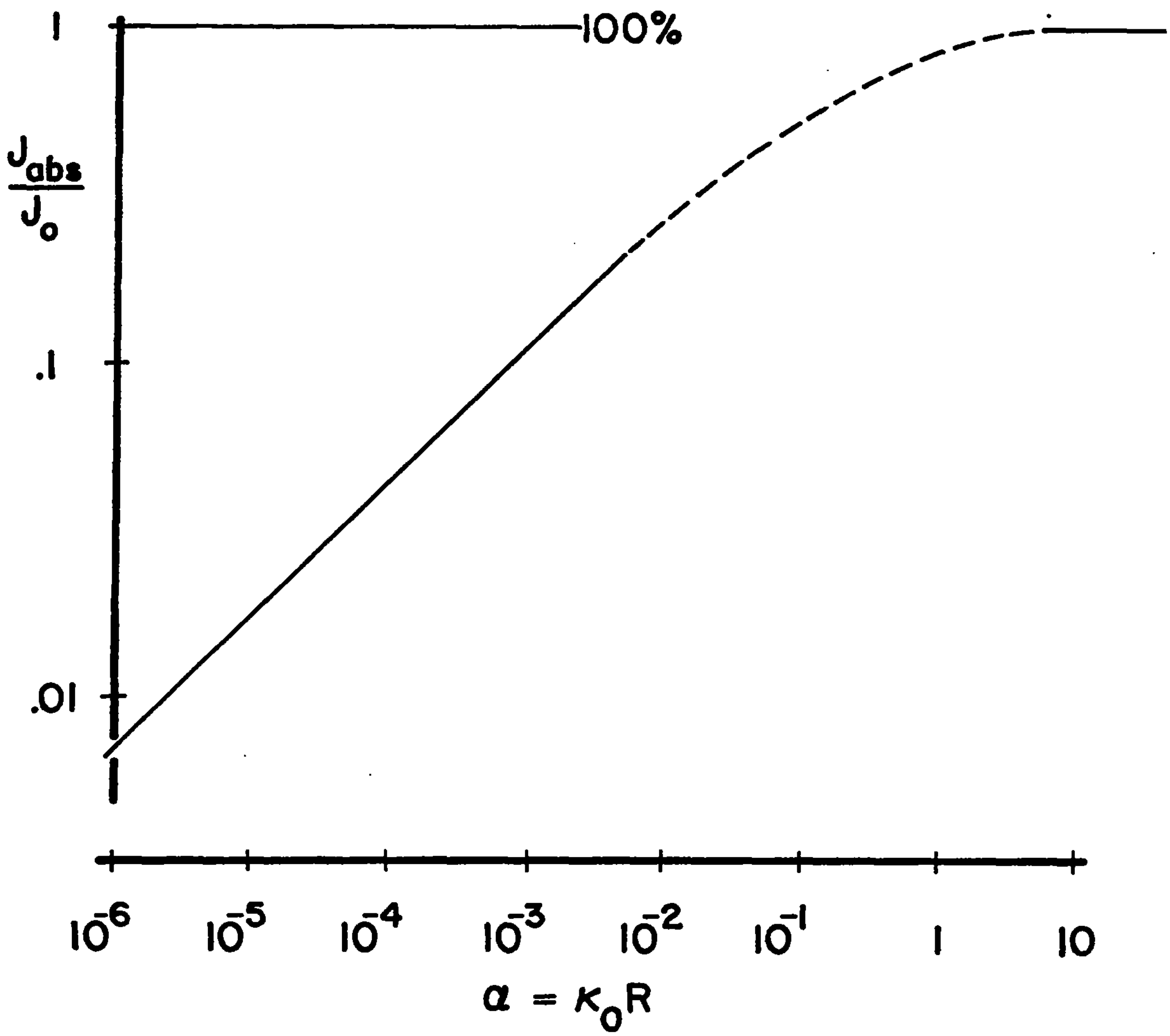

Figure 13. Fraction of Laser Energy Absorbed for a Linear Density Gradient Plasma

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